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STOCHASTIC PARAMETRIC SYSTEM IDENTIFICATION APPROACH FOR VALIDATION OF FINITE ELEMENT MODELS: INDUSTRIAL APPLICATIONS

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Abstract. Stochastic parametric system parameters identification approach with taking into account the aliasing problem for validation of finite element models is presented. Investigated measurement noise perturbation influences to the identified system modal and physical parameters. Estimated measurement noise border, for which identified system parameters are acceptable for validation of finite element model of examine system. System identification is realized by observer Kalman filter and Subspace algorithms. In special case observer gain may be coincide with the Kalman gain. Stochastic state-space model of the structure are simulated by Monte-Carlo method.

Keywords: stochastic systems identification, finite element, validation models, Monte-Carlo simulation, stochastic models.

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1. Introduction

When forced excitation tests are very difficult or only response data are measurable while the actual loading conditions are unknown, operation modal analysis (output-only modal identification techniques) remains the only technique for parametrical identification. The main advantage of this method is that no special, artificial type of excitation has to apply to the structure to determine its dynamic characteristics. Furthermore, if a structure has high period (more than one second) modes, it may be difficult to excite it with a shaker, whereas this is generally no problem for drop weight or ambient sources. However if mass-normalized mode shapes are required, ambient excitation cannot be used. Output-only modal identification techniques efficiently use with model updating tools to develop reliable finite element models of structures.

The term system identification as research field in automatic control was coined by Lutfi Zadeh (1962) [45].

System identification is the process of developing or improving a mathematical representation of a physical system using experimental data investigated in Kalman [10, 17], Ibrahim [11, 12], Bendat [5], Ljung [31], Juang [16], Van Overschee and De Moor [41] and system identification applications in civil engineering structures are presented in works Trifunac [38], Link [30], Ventura [42], Brincker [6], Roeck [8, 44], Peeters [34], Cunha [43], Wenzel [7], Kasimzade [20]. Extracting system physical parameters from identified state space representation was investigated in references [3, 4, 13-15, 32, 35-37, 39, 40]. The solution of a matrix algebraic Riccati equation and orthogonality projection more intensively and inevitably used in system identification was deeply investigated in works of Aliev, F.A., Bordyug, B.A., Larin, V.B. [1, 2].

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In engineering structures there are three types of identification: modal parameter identification; structural-modal parameter identification; control-model identification methods are used.

In the frequency domain the identification is based on the singular value decomposition of the spectral density matrix and it is denoted Frequency Domain Decomposition (FDD) and its further development Enhanced Frequency Domain Decomposition (EFDD).

In the time domain there are three different implementations of the Stochastic Subspace Identification (SSI) technique: Unweighted Principal Component (UPC); Principal component (PC); Canonical Variety Analysis (CVA) are used.

For the modal updating of the structure [9, 21, 22, 33] it is necessary to estimate sensitivity of reaction of examined system to change of random [23, 25-29] or fuzzy [25] parameters of a building.

Below stochastic parametric system parameters identification approach with take into account the aliasing (bound checking) problem for validation of finite element models is presented. Investigated measurement noise perturbation influences to the identified system modal and physical parameters. Estimated measurement noise border, for which identified system parameters are acceptable for validation of finite element model of examine system. System identification is realized by observer Kalman filter [14] and Subspace [41] algorithms. In special case observer gain may be coincide with the Kalman gain. Stochastic state-space model of the structure are simulated by Monte-Carlo method.

2. Time-domain model of the system

In presented paper mentioned problem was investigated for multi degree of freedom structural (buildings, towers, matches and et cetera) systems with no limited number of elements (such as beam, membrane, shell, solid and et cetera). As known for similar type systems the system matrices [m], [c], [k] may be build only by FEM and the equation of motion for a finite-dimensional linear-dynamic system a set of n_2 second-order differential equations are arranged as

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = [d]\{f_{\oplus}(t)\}. \tag{1}$$

Here the direct stiffness method was used for implementation FEM [18, 19, 23] and appropriately was build system mass, damping and stiffness matrices ($[m]$, $[c]$, $[k]$). For example FEM implementation system stiffness matrix [k] by the direct stiffness method shown below as follows [18, 19]:

$$[\bar{k}_r]->[\bar{\bar{k}}_r]=[C_r][\bar{k}_r][C_r]^T->[\bar{\bar{k}}_{r+}]=[\tau_r]^T[\bar{\bar{k}}_r][\tau_r]->[k_\bullet]=\sum_{r=1}^{r_*}[\bar{\bar{k}}_{r+}]->a.b.c.->[k], \tag{1b}$$

here, $[\bar{k}_r]$ is the element stiffness matrix in local coordinate system (c.s.) for $r-$th finite element,
$[\bar{\bar{k}}_r]$ is the element stiffness matrix in global coordinate system for $r-$th finite element,
$[C_r]$ is the coordinate transformation matrix from local to global c.s. for $r-$th finite element,
$[\tau_r]$ is the topology matrix for $r-$th finite element, $a.b.c.$ is abbreviation "mean after application of boundary conditions", r_* is a number of identical finite elements examined system,
$[k]$ is the stiffness matrix of the in examined system in global c.s.
Here main relations of FEM are given on base Lagrange variation principle.

The equation of motion (1) are transformed to the state-space former of first order equations- i.e., a continuous-time state-space model of the system are evaluated as

$$\{\dot{z}(t)\} = [A_c]\{z(t)\} + [B_c]\{f_{\oplus}(t)\}. \tag{2a}$$

$$[A_c] = \begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix}, \tag{2b}$$

$$[B_c] = \begin{bmatrix} [0] \\ [m]^{-1}[d] \end{bmatrix}, \tag{2c}$$

$$\{z(t)\} = \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix}. \tag{2d}$$

If the response of the dynamic system is measured by the m_1 output quantities in the output vector $\{y(t)\}$ using sensors (such as accelerometers, velocity, displacements, etc.,), for system model represented by the equations (2), appropriate measurement-output equation become as

$$\{y(t)\} = [C_a]\{\ddot{u}\} + [C_v]\{\dot{u}\} + [C_d]\{u\} = [C]\{z(t)\} + [D]\{f_{\oplus}(t)\}, \tag{3a}$$

$$[C] = [\ [C_d] - [C_a][m]^{-1}[k], \quad [C_v]\ - [C_a][m]^{-1}[c]\ ], \tag{3b}$$

$$[D] = [C_a][m]^{-1}[d]. \tag{3c}$$

Where $\{u\}$ is the vector of displacement; $[A_c]$, is an n_1 ($n_1 = 2n_2$; n_2 is the number of independed coordinates) by n_1 state matrix ; $[d]$ is an n_2 by r_1 input influence matrix, characterizing the locations and type of known inputs $\{f_{\oplus}(t)\}$; $[C_a]$, $[C_v]$, $[C_d]$ are output influence matrices for acceleration, velocity, displacement for using sensors (such as accelerometers, tachometers, strain gages, etc.,) respectively; $[C]$ is an $m_1\, x\, n_1$ output influence matrix for the state vector $\{z\}$ and displacement only; $[D]$ is an $m_1\, x\, r_1$ direct transmission matrix; r_1 is the number of inputs; m_1 is the number of outputs.

In the output - only modal analysis environment, the main assumption is that input force $\{F(t)\} = [d]\{f_{\oplus}(t)\}$ comes from white noise or time impulse excitation. Under this hypothesis discrete-time stochastic state–space model may be written as:

$$\{z_{k+1}\} = [A]\{z_k\} + [B]\{f_{\oplus k}\} + \{w_k\}, \tag{4}$$

$$\{y_k\} = [C]\{z_k\} + [D]\{f_{\oplus k}\} + \{v_k\}, \tag{5}$$

where $\{z_k\} = \{z(k\Delta t)\}$ is the discrete-time state vector; is the process noise due to disturbance and modeling imperfections; $\{v_k\}$ is the measurement noise due to sensors' inaccuracies; $\{w_k\}, \{v_k\}$ vectors are non-measurable, but assumed that they are white noise with zero mean. If this white noise assumption is violated, in other words if the input contains also some dominant frequency components in addition to white noise, these frequency components cannot be separated from the eigen frequencies of the system and they will appear as eigenvalues of the system matrix $[A]$.

In the real structures, exited by ambient vibration, the input $\{f_{\oplus}(t)\}, \{f_{\oplus k}\}$ remains unmeasured and therefore it disappears from the equations (2)-(5) respectively. Then to take into consideration this fact, the input is implicitly modeled by the noise terms $\{\underline{w}_k\}$, $\{\underline{v}_k\}$, which are indirectly contain no measurable input from ambient vibration and mentioned relation became as:

$$\{z_{k+1}\} = [A]\ \{z_k\} + \{\underline{w}_k\}, \tag{6}$$

$$\{y_k\} = [C]\ \{z_k\} + \{\underline{v}_k\}\,. \tag{7}$$

3. Solution of the deterministic problem described by equation (2a), (3a)

Assuming zero initial conditions $z_{k=0} = 0$, the set of equations (2a,3a) for sequence of $k = 0, 1, 2, \ldots, \ell$ can be grouped in a matrix form (linear difference or ARX model) as

$$[y] = [\mathbf{Y}][\mathbf{F}_{\oplus}]. \tag{8}$$

For the special case where $[A]$ is asymptotically stable so that for some sufficiently large p, $[A]^k \approx 0$ for all time steps $k \geq p$; truncating $[\mathbf{Y}]$**,** $[\mathbf{F}_{\oplus}]$ choose the data length ℓ greater than $r_1(p+1)$ such that $[C][A]^k[B] \approx 0$ for $k \geq p$, equation (8) can be approximated by

$$[y] = \ [Y\,][F_{\oplus}], \tag{9}$$

$$[y] = [y_0\ \ y_1\ \ y_2\ \ \cdots\ \ y_p\ \ \cdots\ \ y_{\ell-1}],$$

$$[Y] = [[D]\quad [C][B]\quad [C][A][B]\ \ \ldots\ \ [C][A]^{p-1}[B]],$$

$$[F_{\oplus}] = \begin{bmatrix} f_{\oplus 0} & f_{\oplus 1} & f_{\oplus 2} & \cdots & f_{\oplus p} & \cdots & f_{\oplus \ell-1} \\ & f_{\oplus 0} & f_{\oplus 1} & \cdots & f_{\oplus p-1} & \cdots & f_{\oplus \ell-2} \\ & & f_{\oplus 0} & \cdots & f_{\oplus p-2} & \cdots & f_{\oplus \ell-3} \\ & & & \cdots & & \cdots & \\ & & & & f_{\oplus 0} & \cdots & f_{\oplus \ell-p-1} \end{bmatrix}$$

and its solution $[Y]$ (the first p Markov parameters) can be approximately determined from the equation (9) as

$$[Y] = [y]\,[F_{\oplus}]^{\dagger}, \tag{10}$$

where matrices dimensions are respectively as $[y](m_1\,x\,\ell)$, $[Y](m_1\,x\,r_1(p{+}1))$, $[F_{\oplus}](r_1(p{+}1)\,x\,\ell)$; $[F_{\oplus}]^{\dagger}$ is the pseudo-inverse of the matrix $[F_{\oplus}]$, and the approximation error decreases as p is increases; ℓ is the number of data samples.

In equation (10) more equations $(m_1\,x\,\ell)$, than unknowns $((m_1\,x\,r_1(p+1))$, because $\ell > r_1(p+1)$.

Unfortunately, for lightly damped space structures, the integer p and thus the number of data samples ℓ required to make the approximation in Eq. (9) valid becomes impractically large in the sense that the size of matrix $[F_{\oplus}]$ is too large to solve the its pseudo-inverse $[F_{\oplus}]^{\dagger}$ numerically.

4. Solution of the stochastic problem described by equations (4), (5) or (6), (7) using observer gain the **okid** algorithm

When using real data with noise, a feedback loop (observer gain $[G]$) is added to artificially increase the damping of the system to make the system (4), (5) or (6), (7) as stable as desired and relation (9) became as

$$[y] = [\bar{\mathbf{Y}}][\mathbf{V}] \tag{11a}$$

which is the input–output description in matrix form for equation:

$$\{z_{k+1}\} = [\bar{A}]\{z_k\} + [\bar{B}]\{\vartheta_k\}, \tag{11b}$$

$$[\bar{A}] = [A] + [G][C],$$

$$[\bar{B}] = [[B] + [G][D] \quad\ - [G]],$$

$$\{\vartheta_k\} = \begin{bmatrix} \{f_{\oplus k}\} \\ \{y_k\} \end{bmatrix}.$$

For the special case where $[\bar{A}]$ is asymptotically stable so that for some sufficiently large p, $[\bar{A}]^k \approx 0$ for all time steps $k \geq p$; truncating $\left[\bar{\mathbf{Y}}\right], [\mathbf{V}]$ choose the data length ℓ greater than $r_1(p+1)$ such that $[C][\bar{A}]^k[\bar{B}] \approx 0$ for $k \geq p$, equation (11a) can be approximated by

$$[y] = [\bar{Y}][V], \tag{12}$$

$$[y] = \begin{bmatrix} y_0 & y_1 & y_2 & \cdots & y_p & \cdots & y_{\ell-1} \end{bmatrix},$$

$$\left[\bar{Y}\right] = \begin{bmatrix} [D] & [C]\left[\bar{B}\right] & [C]\left[\bar{A}\right]\left[\bar{B}\right] & \ldots & [C]\left[\bar{A}\right]^{p-1}\left[\bar{B}\right] \end{bmatrix},$$

$$[V] = \begin{bmatrix} f_{\oplus 0} & f_{\oplus 1} & f_{\oplus 2} & \cdots & f_{\oplus p} & \cdots & f_{\oplus \ell-1} \\ & \vartheta_0 & \vartheta_1 & \ldots & \vartheta_{p-1} & \ldots & \vartheta_{\ell-2} \\ & & \vartheta_0 & \ldots & \vartheta_{p-2} & \ldots & \vartheta_{\ell-3} \\ & & & \cdots & & \cdots & \\ & & & & \vartheta_0 & \ldots & \vartheta_{\ell-p-1} \end{bmatrix}$$

and its (12) solution $[\bar{Y}]$ (the first p Markov parameters) can be approximately determined from real data ($[V]$) as

$$[\bar{Y}] = [y][V]^{\dagger}, \tag{13}$$

where matrices dimensions are respectively as $[y](m_1\, x\, \ell)$, $[\bar{Y}]\,(m_1\, x\, [(r_1+m_1)p+r_1]$, $[V\,]([r_1+m_1)p+r_1]\, x\, \ell)$; $[V]^{\dagger}$ is the pseudo-inverse of the matrix $[V]$, and the approximation error decreases as p is increases. The maximum value of p is the number that maximizes the number $(r_1+m_1)\,p+r_1 \leq \ell$ of independent rows of $[V]$. The maximum p means the upper bound of the order of the deadbeat observer. The lower bound of the order of the observer chosen such that minimum value of p is the number that minimizes the number $m_1\, p \geq n_1$. Consequently for the asymptotically stable solution of Eq. (12), bound of the order of observer p must be in the following interval

$$[m_1\,/\,n_1\,] \leq p \leq [(\ell - r_1)/(r_1+m_1)]. \tag{14}$$

The least squares solution of the Eq. (12) using the batch (non-recursive) algorithm for a sequence $k = 0\ , \ \ldots, \ \ell-1$, assume zero initial conditions, $z_{k=0} = 0$, from Eq. (11b), it is easy to show that

$$[\bar{y}] = [C][\bar{A}]^p[z] \,+\, [\bar{Y}][\bar{V}]. \tag{15}$$

For the case where $[\bar{A}]^p$ is sufficiently small and all the states in $[z]$ are bounded, Eq. (15) can be approximated by neglecting the first term on the right hand side,

$$[\bar{y}] \,= [\bar{Y}][\bar{V}] \tag{16}$$

which has the following least – squares solution for the observer Markov parameters ($[\bar{Y}]$):

$$\begin{aligned} &[\bar{Y}] = [\bar{y}][\bar{V}]^{\dagger}, \\ &[\bar{V}]^{\dagger} = [\bar{V}]^T[[\bar{V}][\bar{V}]^T]^{-1}, \end{aligned} \tag{17}$$

where $[\bar{y}], [\bar{Y}], [\bar{V}]$ are described in the form

$$[\bar{y}] = [y_p,\, y_{p+1},\, \ldots y_{\ell-1}],$$

$$[\bar{Y}] = [\bar{Y}_0,\, \bar{Y}_1,\, \bar{Y}_2,\, \ldots \bar{Y}_{p-1}].$$

Theoretically $[\bar{Y}]$ contains system matrices $[\bar{A}]$, $[\bar{B}], [C]$, $[D]$ so

$$\bar{Y} = [[D] \quad [C][\bar{B}] \quad [C][\bar{A}][\bar{B}] \ \dots \ [C][\bar{A}]^{p-1}[\bar{B}]], \tag{18}$$

$$\bar{V} = \begin{bmatrix} f_{\oplus p} & f_{\oplus p+1} & \cdots & f_{\oplus \ell-1} \\ \vartheta_{p-1} & \vartheta_p & \dots & \vartheta_{\ell-2} \\ \vartheta_{p-2} & \vartheta_{p-1} & \dots & \vartheta_{\ell-3} \\ \dots & \dots & \dots & \dots \\ \vartheta_0 & \vartheta_1 & \dots & \vartheta_{\ell-p-1} \end{bmatrix}. \tag{19}$$

The Eq. (2a, 3a) for the state space observer model (another word for direct measurement) in discrete time domain has the observer form

$$\{\hat{z}_{k+1}\} = [\bar{A}]\{\hat{z}_k\} + [\bar{B}]\{\vartheta_k\} \tag{20}$$

or

$$\{\hat{z}_{k+1}\} = [A]\{\hat{z}_k\} + [B]\{f_{\oplus k}\} - [G](\{y_k\} - \{\hat{y}_k\}), \tag{21}$$

$$\{\hat{y}_k\} = [C]\{\hat{z}_k\} + [D]\{f_{\oplus k}\} \tag{22}$$

with the state estimation error

$$\{e_{k+1}\} = [\bar{A}]\{e_k\},$$

$$\{e_k\} = \{z_k\} - \{\hat{z}_k\}.$$

Where $\{\hat{z}_k\}$ is an observer state vector and $\{\hat{y}_k\}$ is the estimated output.

If $[\bar{A}]$ is asymptotically stable, then for large k, the estimated $\{\hat{z}_k\}$ tends to the true state $\{z_k\}$. Theoretically, one would choose the gain matrix $[G]$ to make state estimation error diminish as quickly as possible. Under ideal conditions, the quickest observer (gain $[G]$) is the Kalman filter ($[K]$).

The initial conditions have negligible influence on the measured data after p time steps. When there are system and measurement noise present, the elimination of initial condition dependence makes the system response become stationary.
Schematic illustration of the Observer gain ($[G]$) identification [25] is presented below:

$$\begin{aligned} &(u, y) \to \bar{y} = \bar{Y}\bar{V} \to \bar{Y} = \bar{y}\bar{V}^{\dagger};\ \bar{Y} \to (OKID) \to Y^0, Y;\ Y \to Y_{pulse} \to \\ &\to ERA/DC[H(Y), SVD(H)] \to A, B, C, D;\ G(A, C, Y^0). \end{aligned} \tag{23}$$

Here H is the Hankel matrix composed from the Markov parameters (Y); SVD(H) is the singular value decomposition of Hankel matrix (H); abbreviation $\bar{Y} \to (OKID) \to Y^0, Y$ is mean: Observer Kalman filter identification algorithm is applied to solution ($\bar{Y}$ - observer Markov parameters) of the equation (16) for separation of observer gain Markov parameters (Y^0) and system Markov parameters (Y) respectively; abbreviation $Y \to Y_{pulse} \to ERA/DC[H(Y),$ $SVD(H)] \to A, B, C, D;\ G(A, C, Y^0)$ is mean: from system Markov parameters (Y) are build pulse response Markov parameters (Y_{pulse}), then applied to system Markov parameters (Y) Eigen Realization Algorithm using Data Correlations (ERA/DC) which contain building of Hankel matrix (H) from system Markov parameters and then by the singular value decomposition of Hankel matrix (SVD(H)), system matrices (A, B, C, D) and gain matrix (G) are build.

5. Kalman filter application

Another (beyond the observer gain including as presented above) way to stochastically characterize system uncertainties including process (input) and measurement (output) noises (described by the Eq. (4), (5) to specify the Kalman filter equation with its steady state Kalman gain ($[K]$), which is function of the process and measurement noise covariances ($[Q], [R]$):

$$\bar{w}_k = E(\{w_k\}) = 0; E(\{w_k\}\{w_j\}^T) = [Q]\,\delta(k-j), \tag{24a}$$

$$\bar{v}_k = E(\{v_k\}) = 0; E(\{v_k\}\{v_j\}^T) = [R]\,\delta(k-j). \tag{24b}$$

Where $E(\ )$ is the expected value operation.
The process noise includes system uncertainties and input noise. Expression for the steady-state Kalman filter model is

$$\{\hat{z}_{k+1}\} = [A]\{\hat{z}_k\} + [B]\{f_k\} + [K](\{y_k\} - \{\hat{y}_k\}), \tag{25}$$

$$\{\hat{y}_k\} = [C]\{\hat{z}_k\} + [D]\{f_k\}. \tag{26}$$

Substituting of Eq. (26) into Eq. (25) yields the following

$$\{\hat{z}_{k+1}\} = [\tilde{A}]\{\hat{z}_k\} + [\tilde{B}]\{\vartheta_k\}. \tag{27}$$

Where $\{y_k\}$ is the real measurement and $\{\hat{y}_k\}$ is the estimated measurement and

$$\left[\tilde{A}\right] = [A] - [K]\,[C]\,,$$

$$\left[\tilde{B}\right] = \left[\ [B] - [K]\,[D] \quad [K]\ \right],$$

$$\{\vartheta_k\} = \begin{bmatrix} \{f_{\oplus k}\} \\ \{y_k\} \end{bmatrix}. \tag{28}$$

The measurement equation becomes

$$\{y_k\} = [C]\,\{\hat{z}_k\} + [D]\,\{f_{\oplus k}\} + \{\varepsilon_k\}\,. \tag{29}$$

A combination of Eqs. (27) and (29) can be written in the following matrix form:

$$\{\bar{y}\} = [C]\{\tilde{A}\}^p[\hat{z}] + [\varepsilon] + [\tilde{Y}][\bar{V}]. \tag{30}$$

Where $[\bar{y}]$ and $[\bar{V}]$ are defined as in Eq. (15)

$$[\bar{y}] = [y_p \;\; y_{p+1} \;\; ... \;\; y_{\ell-1}], \tag{31a}$$

$$[\hat{z}] = [\hat{z}_0 \;\; \hat{z}_1 \;\; ... \;\; \hat{z}_{\ell-p-2}], \tag{31b}$$

$$[\tilde{Y}] = [[D] \quad [C][\tilde{B}] \quad [C][\tilde{A}][\tilde{B}] \;\; ... \;\; [C][\tilde{A}]^{p-1}[\tilde{B}]], \tag{31c}$$

$$[\varepsilon] = \left[\ \varepsilon_{r,p} \quad \varepsilon_{r,p+1} \quad \varepsilon_{r,p+2} \quad ... \quad \varepsilon_{r,\ell-1}\ \right]. \tag{31d}$$

If the observer ($[G]$) happens to be a Kalman filter ($[K]$), then the residual ($[\varepsilon]$) is white, zero-mean, and Gaussian.

The choice of p in Eq. (30) has to be sufficiently large that the transients of the Kalman filter are negligible, i.e.,

$$[C][\tilde{A}]^p[\hat{z}] \approx 0. \tag{32}$$

For case $\ell \to \infty$, for all $k > p$ Eq. (30) can now be written as

$$[\bar{y}] = [\tilde{Y}][\bar{V}]. \tag{33}$$

Solution of the Eq. (33) is

$$[\tilde{Y}] = [\bar{y}][\bar{V}]^{\dagger}. \tag{34}$$

Where

$$[\bar{V}]^{\dagger} = [\bar{V}]^T[\ [\bar{V}][V]^T\ ]^{-1}. \tag{35}$$

We conclude that any observer ($[\bar{Y}]$) satisfying Eq. (15) or its equivalent (30), produces the same input-output map as a Kalman filter does if the data length (ℓ) is sufficiently long and the order (p) of the observer is sufficiently large so that the truncation error is negligible. Therefore, when reduced to system order, the identified observer (gain $[G]$) has to be a Kalman filter (gain $[K]$) and thus the $[G]$ computed from the combined Markov parameters gives the steady-state Kalman filter gain

$$[K] = -[G]. \tag{36}$$

Schematic illustration of the Kalman gain ($[K]$) identification [25] is presented below:

$$\begin{aligned}&(u, y) \to \bar{y} = \tilde{Y}\bar{V} \to \tilde{Y} = \bar{y}\bar{V}^{\dagger};\ \tilde{Y} \to (OKID) \to Y^0, Y;\ Y \to Y_{pulse} \to \\ &\to ERA/DC \to A, B, C, D;\ K(A, C, P, Q, R).\end{aligned} \tag{37}$$

Explanation of abbreviations in (37) similer to (23) except that ($\tilde{Y}$) is solution of the equation (33); Abbreviation $K(A, C, P, Q, R)$ is mean that Kalman gain (K) are build as function of system matrices (A,C), solution (P) of Riccati equation, and the covariances ($[Q], [R]$) respectively of the process and measurement noises. Here Kalman gain ($[K]$) of the building parametrical model Eq. (25), (26) or (27) obtained as

$$[K] = [A][P][C]^T([R] + [C][P][C]^T)^{-1}. \tag{38}$$

In which

$$[P] = E(\{e_k\}\{e_k\}^T), \quad with\ estimation\ error\ \{e_k\} = \{z_k\} - \{\hat{z}_k\} \tag{39}$$

is the error covariance and obtained as solution of the discrete algebraic Riccati equation

$$[P] = [A][P][A]^T - [A][P][C]^T([R] + [C][P][C]^T)^{-1}[C][P][A]^T + [Q]. \tag{40}$$

The existence of the Riccati equation solution is only possible, if the covariance matrix is positive definite. There are a few proposals in literature to guarantee a solution. But the existing experiences it remains an open problem in large scale stochastic realization theory.

In short, for the known dynamics Eq. (4) and measurement $\{y_k\}$ Eq. (5), with aim to find the best (or optimal) estimate $\{z_k\}$ in the sense that the estimation error $\{e_k\} = \{z_k\} - \{\hat{z}_k\}$ is as small as possible, the error covariance $[P\,]$ must be satisfy discrete algebraic Riccati equation (40).

One can obtain Kalman gain ($[K]$) from Eq. (38), (40), for the building parametric model Eq. (4), (5) with (23), (24). Kalman filter equation is evaluated

$$\{\hat{z}_{k+1}\} = [A]\{\hat{z}_k\} + [B]\{f_k\} + [K]\{\varepsilon_k\} \tag{41}$$

with the output measurement y_k satisfying

$$\{y_k\} = [C]\{\hat{z}_k\} + [D]\{f_k\} + \{\,\varepsilon_k\} \tag{42}$$

or

$$\{\hat{y}_k\} = [C]\{\hat{z}_k\} + [D]\{f_k\}.$$

The output residual ($\{\varepsilon_k\} = y_k - \hat{y}_k$) satisfies

$$\{\varepsilon_k\} = [C]\{\, e_k\} \,+ \{\, v_k\}. \tag{43}$$

Comparing building system modeling (Eqs. (4), (5) with (23), (24)) by the Kalman filter equations (41), (42), (43) the error is evaluated as:

- the state estimation error

$$\{e_k\} = \{z_k\} - \{\hat{z}_k\}; \quad E(\{e_k\}) = 0, \tag{44}$$

- error dynamics (the estimation error at steady-state reduces to)

$$\{e_{k+1}\} = \{z_{k+1}\} - \{\hat{z}_{k+1}\} = ([A] - [K][C])\{e_k\} - \,[K]\{v_k\} \,+ \{w_k\}, \tag{45}$$

- output residual

$$\{\varepsilon_k\} = [C]\{\, e_k\} \,+\, \{v_k\}\ ; \quad E(\{\varepsilon_k\}) \,= 0. \tag{46}$$

The Kalman filter is the only optimal state estimation which can be optimal for linear systems. Theoretically, the Kalman filter is very attractive, because it has a closed-form solution (i.e., Riccati equation) for its gain matrix. However, the Kalman filter requires information including the covariances ($[Q], [R]$) respectively of the process and measurement noises. The measurement noise ($\{v_k\}$) may be quantified by a sufficiently large number of repeated tests on the sensors. However, the process noise ($\{w_k\}$) due to modeling error and system uncertainties is very difficult, if not impossible, to quantify in practice. It may by easier to estimate the Kalman filter gain directly from the experimental data (observer gain $[G]$) without estimating ($[Q], [R]$) and solving the Riccati equation.

In practice, due to the presence of the presence of other factors such as disturbances, nonlinearities, and non whiteness of the process and measurement noises, the resultant identified filter is not the Kalman filter.

In such a case, the identified filter is simply an observer ($[G]$) that is computed from input-output data that minimize the filter residual in a least-squares sense.

6. Extracting modal parameters from identified state space model

After obtaining discrete time system matrices $[A], [B], [C], [D]$ by the ERA / OKID based approach [6, 8, 13-15, 37], and these matrices wore converted to their continuous time counter parts

$$[A_C], [B_C], [C_C], [D]\,, \tag{47}$$

eigenvalues $[\Lambda\ ]$ and eigenvectors $[\underline{\psi}]$ of the continuous time system matrix $[A_C]$ is calculated as

$$([\underline{\psi}], [\Lambda] = eig([A_C]). \tag{48}$$

The transformation matrix $[\tau]$ using orthogonality conditions [28] is obtained as

$$[\tau] = \begin{bmatrix} \tau_1 & & & & \\ & \tau_2 & & & \\ & & \dots & & \\ & & & \tau_i & \\ & & & & \dots \end{bmatrix}, \tag{49}$$

$$\tau_i = \sqrt{([\underline{\psi}]^{-1}[B_C^E(:,i)])^T([C_C^E(i,:)][\underline{\psi}])^{-1}}.$$

The mode shape $[\psi(:,r)]$ of the r^{th} mode at the sensor locations are the observed parts of the system eigenvectors $[\underline{\psi}(:,r)]$ of $[\underline{\psi}]$, given by the formal equation:

$$[\psi(:,r)] \ = [C_C][\underline{\psi}(:,r)]. \tag{50}$$

The mode shape $[\psi\ ]$ more detailed calculated by the following Eqs. (51)-(54):
a) If there is a sensor at the k^{th} DOF

$$[\psi(k,:)] = [C_C^E(k,:)][\underline{\psi}][\tau], \tag{51}$$

b) If k^{th} DOF is instrumented with either a sensor or actuator

$$[\psi(k,:)] = ([\tau]^{-1}[\underline{\psi}]^{-1}[B_C^E(:,k)])^T, \tag{52}$$

c) If there is a full set of sensors,

$$[\psi] = [C_d]^{-1}[C_C][\underline{\psi}][\tau], \tag{53}$$

d) If there is a full set of actuators,

$$[\psi]^T = ([\tau]^{-1}[\underline{\psi}]^{-1}[B_C][d]^{-1}. \tag{54}$$

Here $[C_C^E], [B_C^E]$ expended to incorporate all the degrees of freedom of the continuous-time system matrices $[C_C], [B_C]$; $[d]$ is an input influence matrix, characterizing the locations and type of known inputs $\{f\,(t)\}$; $[C_a]$, $[C_v]$, $[C_d]$ are output influence matrices for acceleration, velo- city, displacement for using sensors (such as accelerometers, tachometers, strain gages, etc.,) respectively; using eigenvalues $[\Lambda]$ from eq.(50), (48) for continuous time system

$$[\Lambda] = \begin{bmatrix} \lambda_1 & & & & \\ & \lambda_2 & & & \\ & & \dots & & \\ & & & \lambda_r & \\ & & & & \dots \end{bmatrix}, \tag{55}$$

$$\lambda_r = c_r + i\omega_r,$$

$c_r = Re(\lambda_r)$ is the damping factor,
$\omega_r = Im(\lambda_r)$ is the damped natural frequency.

The damping ration ξ_r of the r^{th} mode is given by

$$\xi_r = -\frac{c_r}{\sqrt{c_r^2 + \omega_k^2}}. \tag{56}$$

Above obtained system modal parameters $[\psi], [\Lambda]$ from the input-output measurements, theoretically must be satisfies the complex eigenvalue problem with known physical parameters $[m], [c], [k]$ as:

$$(\lambda_r^2[m] + \lambda_r[c] + [k])\{\psi_r\} = 0; \tag{57}$$

Here

$$[\Lambda] = \begin{bmatrix} \lambda_1 & & & & \\ & \lambda_2 & & & \\ & & ... & & \\ & & & \lambda_r & \\ & & & & ... \end{bmatrix},$$

$$\lambda_r = c_r + i\omega_r; i = \sqrt{-1},$$

$$[\psi] = [\{\psi_1\} \ \ \{\psi_2\} \ ... \ \{\psi_r\} \ ...].$$

7. Extracting physical parameters from identified state space model

The mass $[m]$, damping $[c]$, stiffness $[k]$ matrices of the finite element model can be obtained using orthogonality conditions [4] from the modal parameters as [32, 35, 39, 40]

$$[m] = ([\psi][\Lambda][\psi]^T)^{-1}, \tag{58}$$

$$[c] = -[m][\psi][\Lambda]^2[\psi]^T[m], \tag{59}$$

$$[k] = -([\psi][\Lambda]^{-1}[\psi]^T)^{-1}, \tag{60}$$

$$[\psi][\psi]^T = 0. \tag{61}$$

The minimum requirement for the above representation is that all degrees of freedom should contain either a sensor or an actuator, with at least one co-located sensor-actuator pair. But in general, it is possible to have more co-located sensors and actuators. These extra conditions are redundant if the system is noise free. However, in the presence of noise it might be best to proceed with a least squares approach to obtain the entries of the matrix $[\tau]$.

Below numerical investigation of stochastic parametrical system parameters identification approach with take into account the aliasing (bound checking) problem for validation of finite element models is presented.

8. Numerical investigations

Three degree of freedom system with classical damping was investigated. The analytical system mass, damping, damping rations, stiffness matrices are presented as
Mass matrix

$$[m] = \begin{bmatrix} 181.35 & 0 & 0 \\ 0 & 181.35 & 0 \\ 0 & 0 & 90.674 \end{bmatrix}.$$

Uncoupled (classical-proportional) damping matrix

$$[c] = \begin{bmatrix} 621.25 & -227.5 & 0 \\ -227.5 & 621.5 & -227.5 \\ 0 & -227.5 & 311.5 \end{bmatrix}.$$

Damping rations for uncoupled (classical) damping case

$$\{\xi\} = \begin{Bmatrix} 0.05 \\ 0.05 \\ 0.0593 \end{Bmatrix}.$$

Stiffness matrix

$$[k] = \begin{bmatrix} 213500 & -106750 & 0 \\ -106750 & 213500 & -106750 \\ 0 & -106750 & 106750 \end{bmatrix}.$$

Internal white noise base excitation ($\ddot{\Delta}(t)$) indirectly as $-m_{i,i}\ddot{\Delta}(t)$ is applied at the i=1, 2, 3-th DOFs appropriately , another word for every i=1, 2, 3-th DOFs actuator excitation is $f_{i,i} = -m_{i,i}\ddot{\Delta}(t)$. Related with it, input influence matrix became as

$$[d] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Base acceleration presented as one realization of normally distributed white noise with zero mean value and with standard deviation as $\sigma_{\ddot{\Delta}} = 0.000327\,\mathrm{m/sec}^2$. Assumed that responses are measured as acceleration of the 1, 2, 3-th DOF. Respectively output influence matrices are presented as

$$[C_a] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ [C_v] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ [C_d] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Sampling time (Δ_t) was determined from the following known relation $\Delta_t \leq (\min(T_0))/2$. For presented example $T_0 = [0.50029 \;\; 0.18312 \;\; 0.13405]^{\mathrm{T}}$ are the system natural vibration periods, $\Delta_t \leq (\min(T_0))/2 = 0.13405/2 = 0.067\,\mathrm{sec}$ and sampling time is accepted as $\Delta_t = 0.05$ sec. Total number of sample points is accepted 500.

Effective frequencies of the system $f = 1/T = \omega/2\pi$ must be in interval $0 \div f_{\mathrm{Nyquist}}$. Where f_{Nyquist} is the Nyquist Frequency which presented by the relation as $f_{\mathrm{Nyquist}} = \frac{1}{2\Delta_t}$. It is known aliasing problem: Standard way of solving is to use analogy low-pass filters with a cut-off frequency of about $0.8 f_{\mathrm{Nyquist}}$. For presented example

$$f_{system\ effective} = 0 \div f_{\mathrm{Nyquist}} = 0 \div \frac{1}{2\Delta_t} = 0 \div 10.$$

Standard way of solving

$$f_{system\ effective\ accepted} = 0 \div 0.8 f_{\mathrm{Nyquist}} = 0 \div 0.8 * 10 = 0 \div 8.$$

Appropriately system effective period must be in interval

$$T_{system\ effective\,accepted} = \infty \div 0.125\,\mathrm{sec}.$$

As seen in presented example, system periods $T_0 = [0.50029\ \ 0.18312\ \ 0.13405]^{\mathrm{T}}$ are in above ($T_0 = \infty \div 0.125\,\mathrm{sec}$) interval. The effective system frequency ($\max(f)$) must be less than Nyquist frequency, e.s. $\max(f) < f_{\mathrm{Nyquist}}$. For presented example

$$\max(f_0) = 7.4599 < f_{\mathrm{Nyquist}} = 10.$$

Known free vibration ($[m]\{\ddot{u}(t)\} + [k]\{u(t)\} = [0]$) results:
Eigenvectors

$$\psi_0 = \begin{bmatrix} -3.5355e-1 & -7.0711e-1 & 3.5355e-1 \\ -6.1237e-1 & -6.1811e-16 & -6.1237e-1 \\ -7.0711e-1 & 7.0711e-1 & 7.0711e-1 \end{bmatrix}.$$

Frequencies and periods

$$\{f_0\} = \left\{ \begin{array}{c} 1.9988 \\ 5.4609 \\ 7.4597 \end{array} \right\}, \quad \{T_0\} = \left\{ \begin{array}{c} 0.5003 \\ 0.1881 \\ 0.1341 \end{array} \right\}.$$

8A) Results for deterministic case are:

The discrete time state space model is identified from recorded input /output data using ERA/DC and Subspace identification approaches:
-Identified system modal parameters
Damping ratio

$$\{\xi_e\} = \left\{ \begin{array}{c} 5.0008\mathrm{e}-2 \\ 4.9968\mathrm{e}-2 \\ 5.9758\mathrm{e}-2 \end{array} \right\}.$$

Frequencies and periods

$$\{f_e\} = \left\{ \begin{array}{c} 1.9988 \\ 5.4609 \\ 7.4597 \end{array} \right\}, \quad \{T_e\} = \left\{ \begin{array}{c} 5.0029\mathrm{e}-1 \\ 1.8312\mathrm{e}-1 \\ 1.3405\mathrm{e}-1 \end{array} \right\}.$$

Mode shapes

$$[\psi_e] = \begin{bmatrix} 8.5593e-3 \pm 8.5603e-3i & -51787e-3 \mp 5.1787e-3i & 4.4331e-3 \pm 4.4311e-3i \\ 7.4131e-3 \pm 7.4129e-3i & -9.9420e-7 \pm 9.4915e-7i & -3.8380e-3 \mp 3.8386e-3i \\ 4.2801e-3 \pm 4.2796e-3i & 5.1787e-3 \pm 5.1787e-3i & 2.2155e-3 \pm 2.2165e-3i \end{bmatrix}.$$

Theoretically must be satisfied $[\psi]^*\,[\psi]_\bullet^T = [0]$. In presented example

$$[\psi_e]^*\,[\psi_e]_\bullet^T =$$
$$= \begin{bmatrix} 3.8491e-14+5.4210e-20i & -2.1115e-17+3.3881e-20i & -3.8570e-14-3.3881e-20i \\ -2.1115e-17+3.3881e-20i & -37920e-17-6.7763e-21i & -1.5179e-17-1.0164e-20i \\ -3.8570e-14-3.3881e-20i & -1.5179e-17-1.0164e-20i & 3.8529e-14+1.8635e-20i \end{bmatrix}.$$

where $[\psi]_\bullet^T$ is the conjugated transpose of the matrix $[\psi]$.
Physical parameters:

$$[m_e] = \begin{bmatrix} 1.8135\text{e}+2 & -1.0732\text{e}-12 & 3.6249\text{e}-9 \\ -1.0732\text{e}-12 & 1.8135\text{e}+2 & 4.8082\text{e}-12 \\ 3.6249\text{e}-9 & 4.8046\text{e}-12 & 9.0674\text{e}+1 \end{bmatrix},$$

$$[k_e] = \begin{bmatrix} 2.1350\text{e}+5 & -1.0675\text{e}+5 & 6.8330\text{e}-6 \\ -1.0675\text{e}+5 & 2.1350\text{e}+5 & -1.0675\text{e}+5 \\ 6.8330\text{e}-6 & -1.0675\text{e}+5 & 1.0675\text{e}+5 \end{bmatrix},$$

$$[c_e] = \begin{bmatrix} 6.2125\text{e}+2 & -2.2750\text{e}+2 & -7.3405\text{e}-7 \\ -2.2750\text{e}+2 & 6.2125\text{e}+2 & -2.2750\text{e}+2 \\ -7.3405\text{e}-7 & -2.2750\text{e}+2 & 3.1150\text{e}+2 \end{bmatrix}.$$

Maximum difference between analytical and extracting experimental results for damping ration is 7.7258e-1%, for frequencies is 7.6262e-7%, for system mass matrix is 4.0095e-9 %, for system stiffness matrix is 6.3995e-9 %, for system damping matrix is 2.3615e-7%. As seen they are exactly with analytical system parameters.

For the case 0.01 sec sampling time (which also satisfied known relation for determination sampling time), extracted physical parameters has one order low accuracy relatively to above (0.05 sec sampling time) results, which may matter in aliasing problem, especially in appropriate stochastic investigations.

8B) Results for stochastic case are:

For the input (actuators in the i=1, 2, 3 system DOFs) as one realization of normally distributed white noise with zero mean value and with standard deviation as

$$\sigma_f = [5.966\text{e}-2 \quad 5.9635\text{e}-2 \quad 2.9849\text{e}-2]$$

system not polluted output mean and standard values are obtained as

$$y_{mean} = [5.7057\text{e}-7 \quad 2.7945\text{e}-7 \quad 0.7049\text{e}-7],$$

$$\sigma_y = [5.7595\text{e}-4 \quad 7.0606\text{e}-4 \quad 8.3182\text{e}-4].$$

Stochastic state-space model of the structure are simulated by Monte-Carlo method. The necessary number of the Monte-Carlo realization $\underline{N}_*$ of random base acceleration, such that, with specified probability $\underline{\beta}$, we can expect that the arithmetic mean variable $\underline{S}$ (element of the output state) will depart from its expected value not more than $\underline{\varepsilon}$, determined from the equation

$$\underline{N}_* = (\underline{\sigma}_S/\underline{\varepsilon})^2[\underline{\phi}^{-1}(\underline{\beta}/2)]^2.$$

Where $\underline{\sigma}_S$ is the standard deviation of random variable $\underline{S}$; $\underline{\phi}^{-1}$ is an inverse Laplace function. For examined example $\underline{N}_*$ is obtained equal to 300. For more details applications of above equation see [27, 28, and 29].

For every Monte Carlo realization every story output ($y(:, i)$, $i = 1,\ 2,\ 3$) is polluted with zero mean and standard deviation equal to 0.0005% σ_y= 0.000005σ_y white noise. The signal to noise ratio (1.6616e+005) for presented example must be and is sufficiently high ($>> 15 - 25$). The discrete time state space model is identified from recorded input /output data using OKID/ERA and Subspace identification approaches:

-Identified system modal and physical parameters mean, standard (std) deviations, coefficients of variation (%, cv=(100 σ/mean), absolute values of the percentage errors in the mean values [$error = abs\ [100\ (analytical - experimental\_mean)/\ analytical]$] of the identified samples respectively are:

For modal parameters

$$\xi_{mean} = \left\{ \begin{array}{c} 5.0008\text{e}-2 \\ 4.9967\text{e}-2 \\ 5.9760\text{e}-2 \end{array} \right\}, \xi_{std} = \left\{ \begin{array}{c} 2.6141\text{e}-7 \\ 5.2006\text{e}-7 \\ 1.0597\text{e}-6 \end{array} \right\}, \xi_{cv} = \left\{ \begin{array}{c} 5.2273\text{e}-4 \\ 1.0408\text{e}-3 \\ 1.7733\text{e}-3 \end{array} \right\}, \xi_{error} = \left\{ \begin{array}{c} 6.6079\text{e}-4 \\ 1.8527\text{e}-3 \\ 3.5714\text{e}-3 \end{array} \right\},$$

$$f_{mean} = \left\{ \begin{array}{c} 1.9988 \\ 5.4609 \\ 7.4597 \end{array} \right\}, \ f_{std} = \left\{ \begin{array}{c} 8.0124\text{e}-7 \\ 2.0996\text{e}-6 \\ 8.0541\text{e}-6 \end{array} \right\}, \ f_{cv} = \left\{ \begin{array}{c} 4.0085\text{e}-5 \\ 3.8448\text{e}-5 \\ 1.0797\text{e}-4 \end{array} \right\}, \ f_{error} = \left\{ \begin{array}{c} 1.2464\text{e}-3 \\ 1.6737\text{e}-4 \\ 3.5072\text{e}-4 \end{array} \right\},$$

$$[\psi_{mean}] = \left[\begin{array}{ccc} 1.2839\text{e}-3 \pm 1.2840\text{e}-3\text{i} & -5.9105\text{e}-3 \mp 5.0903\text{e}-3\text{i} & 4.4332\text{e}-3 \pm 4.4312\text{e}-3\text{i} \\ 1.1120\text{e}-3 \pm 1.1119\text{e}-3\text{i} & -9.6471\text{e}-7 \pm 1.1120\text{e}-6\text{i} & -3.8381\text{e}-3 \mp 3.8384\text{e}-3\text{i} \\ 6.4203\text{e}-4 \pm 6.4192\text{e}-4\text{i} & 5.9104\text{e}-3 \pm 5.0904\text{e}-3\text{i} & 2.2153\text{e}-3 \pm 2.2165\text{e}-3\text{i} \end{array} \right],$$

$$[\psi_{mean}]\,[\psi_{mean}]_{\bullet}^{T} =$$
$$= \left[\begin{array}{ccc} 1.8080e-5+4.4723e-19i & 1.0200e-8+3.3881e-21i & -1.8044e-5-4.7095e-19i \\ 1.0200e-8+3.3881e-21i & -44932e-9 & -1.1949e-8+1.6941e-21i \\ -1.8044e-5-4.7095e-19i & -1.1949e-8+1.6941e-21i & 1.8032e-5+4.7942e-19i \end{array} \right].$$

Where $[\psi_{mean}]_{\bullet}^{T}$ is the conjugated transpose of the matrix $[\psi_{mean}]$.

$$[\psi_{std}] = \left[\begin{array}{ccc} 8.4840\text{e}-3 \pm 8.4841\text{e}-3\text{i} & 1.8598\text{e}-4 \pm 9.8615\text{e}-5\text{i} & 1.2230\text{e}-7 \pm 8.8534\text{e}-8\text{i} \\ 7.3479\text{e}-3 \pm 7.3468\text{e}-3\text{i} & 4.3055\text{e}-8 \mp 2.2841\text{e}-8\text{i} & 9.3666\text{e}-8 \mp 3.4518\text{e}-8\text{i} \\ 4.2425\text{e}-3 \pm 4.2415\text{e}-3\text{i} & 1.8597\text{e}-4 \pm 9.8635\text{e}-5\text{i} & 4.0974\text{e}-8 \mp 3.1793\text{e}-10\text{i} \end{array} \right],$$

$$[\psi_{cv}] = \left[\begin{array}{cccccc} 6.6077\text{e}+2 & 6.6077\text{e}+2 & 2.6987 & 2.6987 & 2.4088\text{e}-3 & 2.4088\text{e}-3 \\ 6.6077\text{e}+2 & 6.6077\text{e}+2 & 3.3106 & 3.3106 & 1.8390\text{e}-3 & 1.8390\text{e}-3 \\ 6.6077\text{e}+2 & 6.6077\text{e}+2 & 2.6988 & 2.6988 & 1.3075\text{e}-3 & 1.3075\text{e}-3 \end{array} \right],$$

$$[\psi_{error}] = \left[\begin{array}{cccccc} 8.5\text{e}+1 & 8.5\text{e}+1 & 1.0064\text{e}+1 & 1.0064\text{e}+1 & 1.8450\text{e}-3 & 1.8450\text{e}-3 \\ 8.5\text{e}+1 & 8.5\text{e}+1 & 1.2043\text{e}+1 & 1.2043\text{e}+1 & 3.7207\text{e}-3 & 3.7207\text{e}-3 \\ 8.5\text{e}+1 & 8.5\text{e}+1 & 1.0064\text{e}+1 & 1.0064\text{e}+1 & 5.4063\text{e}-3 & 5.4063\text{e}-3 \end{array} \right].$$

For physical parameters

$$[m_{e\_mean}] = \left[\begin{array}{ccc} 1.6944e+2 & -3.6133e-3 & 5.9564 \\ -3.6133e-3 & 1.8135e+2 & 5.0989e-3 \\ 5.9564 & 5.0989e-3 & 8.7697e+1 \end{array} \right],$$

$$[m_{e\_std}] = \left[\begin{array}{ccc} 7.2182 & 3.3631e-3 & 3.6106 \\ 3.3631e-3 & 5.3259e-3 & 3.9847e-3 \\ 3.6106 & 3.9847e-3 & 1.8045 \end{array} \right],$$

$$[m_{e\_cv}] = \left[\begin{array}{ccc} 4.2600 & 9.3076e+1 & 6.0618e+1 \\ 9.3076e+1 & 2.9368e-3 & 7.8148e+1 \\ 6.0618e+1 & 7.8148e+1 & 2.0577 \end{array} \right],$$

$$[m_{error}] = \left[\begin{array}{ccc} 6.5649 & - & - \\ - & 3.1105\text{e}-3 & - \\ - & - & 3.283 \end{array} \right],$$

$$[k_{e\_mean}] = \left[\begin{array}{ccc} 1.9758\text{e}+5 & -1.0676\text{e}+5 & 7.9647\text{e}+3 \\ -1.0676\text{e}+5 & 2.1350\text{e}+5 & -1.0675\text{e}+5 \\ 7.9647\text{e}+3 & -1.0675\text{e}+5 & 1.0277\text{e}+5 \end{array} \right],$$

$$[k_{e\_std}] = \begin{bmatrix} 8.4465\text{e}+3 & 5.1078 & 4.2246\text{e}+3 \\ 5.1078 & 8.8516 & 5.6436 \\ 4.2246\text{e}+3 & 5.6436 & 2.1116\text{e}+3 \end{bmatrix},$$

$$[k_{e\_cv}] = \begin{bmatrix} 4.2750 & 4.7845\text{e}-3 & 5.3042\text{e}+1 \\ 4.7845\text{e}-3 & 4.1459\text{e}-3 & 5.2869\text{e}-3 \\ 5.3042\text{e}+1 & 5.2869\text{e}-3 & 2.0548 \end{bmatrix},$$

$$[k_{error}] = \begin{bmatrix} 7.4558 & 6.5914\text{e}-3 & - \\ 6.5914\text{e}-3 & 1.3271\text{e}-3 & 2.7317\text{e}-3 \\ - & 2.7317\text{e}-3 & 3.7326 \end{bmatrix},$$

$$[c_{e\_mean}] = \begin{bmatrix} 1.1452\text{e}+3 & -2.2757\text{e}+2 & -2.6194\text{e}+2 \\ -2.2757\text{e}+2 & 6.2166\text{e}+2 & -2.2746\text{e}+2 \\ -2.6194\text{e}+2 & -2.2746\text{e}+2 & 4.4250\text{e}+2 \end{bmatrix},$$

$$[c_{e\_std}] = \begin{bmatrix} 1.8798\text{e}+2 & 9.2429\text{e}-2 & 9.4019\text{e}+1 \\ 9.2429\text{e}-2 & 3.1676\text{e}-1 & 5.3997\text{e}-2 \\ 9.4019\text{e}+1 & 5.3997\text{e}-2 & 4.7030\text{e}+1 \end{bmatrix},$$

$$[c_{e\_cv}] = \begin{bmatrix} 1.6414\text{e}+1 & 4.0616\text{e}-2 & 3.5893\text{e}+1 \\ 4.0616\text{e}-2 & 5.0954\text{e}-2 & 2.3739\text{e}-2 \\ 3.5893\text{e}+1 & 2.3739\text{e}-2 & 1.0628\text{e}+1 \end{bmatrix},$$

$$[c_{error}] = \begin{bmatrix} 8.4338\text{e}+1 & 2.9994\text{e}-2 & - \\ 2.9994\text{e}-2 & 6.6723\text{e}-2 & 1.6926\text{e}-2 \\ - & 1.6926\text{e}-2 & 4.2054\text{e}+1 \end{bmatrix}.$$

As seen for sampling time $\Delta_t = 0.05$ *Second* and (0.0005) output pollution (0.0005% σ_y= 0.000005σ_y) maximum relative error for damping ration is 3.5714e-3 %, for frequencies is 1.2464e-3%, for mode shapes 85% (but satisfies condition $[\psi_{mean}][\psi_{mean}]^T_{\bullet} \approx [0]$), for system mass matrix is 6.5649%, for system stiffness matrix is 7.4558%, for system damping matrix is 84.338%.

Summarize the results, for the case when existing **low-order (no more than** (0.0005% σ_y) **output pollution** algorithm for extracting physical parameters **partly (except of damping matrix) may be used in applications.**

For another case with sampling time $\Delta_t = 0.05$ *Second* and 1% output pollution (1% σ_y= 0.01σ_y) maximum relative error for damping ration is 458.58 %, for frequencies is 6.5%, for mode shapes 1293% (but satisfies condition $[\psi_{mean}][\psi_{mean}]^T_{\bullet} \approx [0]$), for system mass matrix is 493%, for system stiffness matrix is 1469%, for system damping matrix is 1e+5%. As seen for the case 1% **output pollution (**1% σ_y= 0.01σ_y**) algorithm** for extracting modal and physical parameters **is not acceptable for applications.**

9. Industrial applications

The four storey space steel frame structure was build by the Earthquake Engineering Research Laboratory at the University of Ondokuz Mayis (in scope of the research project MF-046 is supported by the University research fond by leading off the author) for the testing with aim comparing several identification techniques (including ambient vibration) and other various structural engineering research studies (Figure 1). It is two-by-two bay, 3.0 m x 5.0 m in plan and 4.6 m in height. Details of the structure are given in [20, 21, and 26]. All of devises with appropriate software and necessary instruments for structural monitoring are placed in mobile vehicle designed in scope of the research project MF-046 and use as mobile structural monitoring system (Figure 1 and website: http://www2.omu.edu.tr/akademikper.asp?id=1528). Response of the structure from ambient vibration is presented in Figure 2 and Figure 3 respectively.

Figure 1. Mobile structural monitoring system and steel frame benchmark structure.

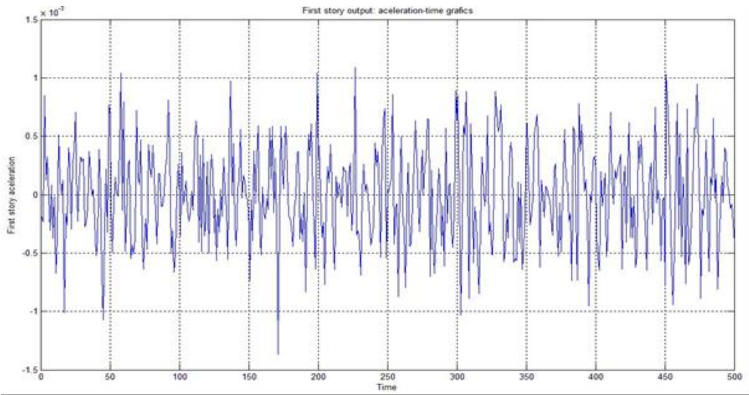

Figure 2. First story output: acceleration-time graphic.

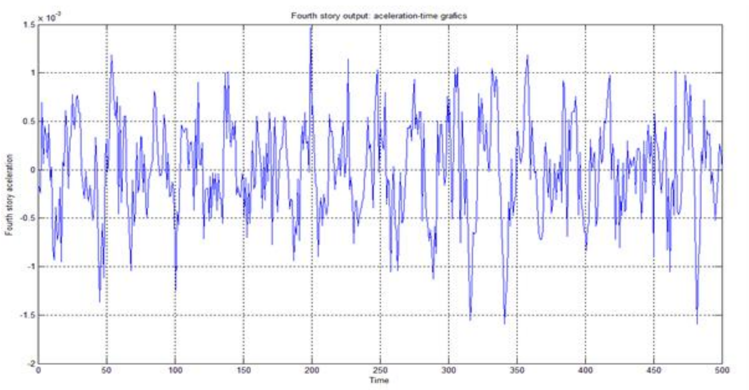

Figure 3. Fourth story output: acceleration-time graphic.

Identified system physical parameters by the relations (58)-(61) are not acceptable. It shown that output noise is more than 0.0005%. As seen for the case more than 0.0005% **output pollution** (0.0005% σ_y**) algorithm** for extracting physical parameters **is not acceptable for applications.** For industrial applications, model updating tools is efficiently use to develop reliable finite element models of structures [9, 21, 22, 33].

10. Conclusions

In this study, stochastic parametric system parameters identification approach with taking into account the aliasing problem for validation of finite element models is presented. It was shown that, existing algorithms for extracting system physical parameters are very sensitive to output noise. In presented example, only for the case when existing low-order (no more than 0.0005% σ_y) output pollution, algorithm for extracting physical parameters partly (except of damping matrix) may be used in applications. When the algorithms are applied to the real instrumented structure, the results are not quite acceptable for extracting physical parameters. In real structures it is very difficult, in many cases it is not possible estimate output pollution percent. But compeering with the academic example, may be estimated that output pollution more than 0.0005% (0.0005% σ_y= 0.000005σ_y).

Otherwise for cases **output pollution** more than 0.0005% **(**0.0005% σ_y**), algorithm** for extracting physical parameters **is not acceptable for applications.** For this case finite element model updating [9, 33] tools is more productive for verification and validation of numerical model in applications [21, 22, and 26].

The aliasing (bound checking) problem for validation of finite element models is presented. In presented example for the case 0.01 sec sampling time, extracted physical parameters has one order low accuracy relatively to 0.05 sec sampling time results, which may matter in aliasing problem, especially in appropriate stochastic investigations. (Both 0.01 sec and 0.05 sec sampling times are satisfied known relation for determination sampling time)

System identification is realized by observer Kalman filter and Subspace algorithms. The Subspace algorithm represents a feasible tool for the identification real partially instrumented structures.

The possibility to improve the implementation of the Least Squares Method algorithm in civil structures to obtain system physical parameters, that allow evaluating the structures behaviour.

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